



Estimating the Optimum Tilt Angles for South-Facing Surfaces in Pristina

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Abstract— Solar energy is derived from photons of light coming from the sun in a form called radiation. Solar energy finds extensive application in air and water heating, solar cooking, as well as electrical power generation, depending on the way of capturing, converting and distribution. To enable such application, it is necessary to analyze the horizontal tilt angle of horizontal surfaces – in order that when the solar energy reaches the earth surface to be completely absorbed. This paper tends to describe the availability of solar radiation for south-facing flat surfaces. The optimal monthly, seasonal, and annual tilt angles have been estimated for Pristina. The solar radiation received by the incident plane is estimated based on isotropic sky analysis models, namely Liu and Jordan model. The annual optimum tilt angle for Pristina was found to be 34.7°. The determination of annual solar energy gains is done by applying the optimal monthly, seasonal and annual tilt angles for an inclined surface compared to a horizontal surface. Monthly, seasonal and annual percentages of solar energy gains have been estimated to be 21.35%, 19.98%, and 14.43%. Losses of solar energy were estimated by 1.13 % when a surface was fixed at a seasonal optimum tilt angle, and when it was fixed at an annual optimum tilt angle, those losses were 5.7%.

Keywords— Solar energy, Gains; Estimation; Tilt angle; South-facing.

I. INTRODUCTION

Because of a significant increase in energy demands, conventional energy sources are being violently consumed, leading to an increase of pollutants released from the burning of fossil fuels. Knowing that solar equipment's do not have moving parts, they are considered to have a greater lifetime and reduce air pollution compared to other energy sources. Therefore, solar energy is considered amongst the most common solutions production thermal energy, for heat water and district heating, and electricity energy.

In most cities of Kosovo, the maximum global solar radiation is reached during July, whereas the minimum during December.

The performance of solar equipment's (i.e. solar collectors for water and air heating, power generation, photovoltaic systems, etc.), is closely related to the inclined angle from the surface that absorbs the light of the sunshine (i.e. with their placement plane). Sun tracking systems are used to increase the acceptance of solar radiation. The usage of these systems has a significant cost to the normal operation of an entire system, due to the consumption of a considerable energy generated.

Hence, the estimation of the optimal tilt angle has a crucial effect on the solar technology. Additionally, trackers need periodic maintenance and calibration and require input energy for their operation which is in the range of 5–10% of the energy produced [1]. Further, trackers are made up of sophisticated mechanical parts which add to capital cost and an increase in cost of absolute power produced from solar PV panels [2]. Other method readily suggested by researchers is to optimize the orientation of flat surfaces at optimum tilt inclination (β_{opt}) [3]. Vieira et al. [4] performed an experimental study which suggested that the sun tracking panel exhibited a low average gain in

power generated, relative to the fixed panel. In another study conducted by Sinha and Chandel [2], it was reported that the horizontal axis weekly adjustment tracking systems and vertical axis continuous adjustment tracking systems, produced less energy annually than the existing PV systems at fixed tilt. Optimum tilt inclination can be adjusted daily, monthly, seasonally, bi-annually or annually for maximizing the performance of the device in use [5].

In the city of Shtërpçë, the annual solar radiation is estimated to be 1333.7 kWh/m²/year, while in the city of Gjakovë 1495.1 kWh/m²/year. Furthermore, knowing the geographical position of the aforementioned cities, we can accept an average value of solar radiation of 1400 kWh/m²/year for the climate conditions of Kosovo [6].

This solar radiation potential can be utilized in desalination, solar-thermal collectors, building heating, day-lighting, and Photovoltaic (PV) Cells, etc. Researchers are therefore concerned to maximize the amount of useful energy that can be extracted through the incoming solar radiations. It is believed that proper installation of these devices can make a remarkable change in the observed performance. Hence, climatology, latitude, orientation, tilt angle, azimuth angles and the usage over a period in a specific geographical region affect the performance of the abovementioned devices [3].

The tilt of a surface (β) is one of the significant factors that considerably affect the availability of solar radiation on a flat surface. Optimization of the performance of solar-based devices requires option-like solar tracking equipment, which follow trajectories of Sun's motion to enhance incident radiation [7], [8]. However, these options are not always economical.

The Optimization of the tilt angle has been conducted by numerous researchers in different countries in Europe [9-14], Middle East [15-25], and Asia [26-32]. In European countries includes studies conducted by Bakirci [9] and Ertekin [10] for Turkey; Stanciu et al. [11] for Romania; Hartner et al. [12] for Austria and Germany; Calabr [13], for Italy; Mehleri et al. [14] for Greece.

1.1. Location under study

Pristina – the capital city – (42.65° N, 21.15° E and 573 m a.s.l.) is situated in the north-east of Kosovo characterized by a humid continental climate with maritime influences. Summers are warm and winters are relatively cold and snowy.

According to a study conducted by the Ministry of Energy and Mining of Kosovo MEM for zones with solar radiation potential, Kosovo has been divided into four zones of rough solar radiation (2010) [6].

Table 1. Division of Kosovo municipalities according to climate sub-zones

Nr	Zone 1	Zone 2	Zone 3	Zone 4
1.	Peje	Prizren	Podujeve	Gjilan
2.	Deçan	Dragash	Kamenice	Vitit
3.	Gjakove	Mitrovice	Istog	Kaçanik
4.	Kline	Skenderaj	Zubin Potok	Shtërpçë
5.	Rahovec	Lipjan	Leposaviq	
6.	Suhareke	Prishtine	Zveçan	
7.	Malisheve	Ferizaj	Vushtrri	

Population density in Kosovo is greater in the central and western parts compared to the eastern parts. Hence, the division into four zones in an acceptable approximation since the solar radiation in Zone 3 and Zone 4 do not change much, as well as the opportunities to widely use solar

energy in Zone 4 (which is smaller regarding surface area) are practically not that greater. The highest values of solar radiation are shown in Zone 1 while the lowest appear in Zone 4. The division of municipalities by area of solar radiation intensity is presented in Table 1 MEM (2010) [6].

II. METHODOLOGY OF ANALYSIS CONCEPTS – DATA ON SOLAR RADIATION

For the purposes of this paper, we used meteorological data for daily mean global radiation and diffuse radiation on a horizontal plane, extracted from the information provided by the Meteorological Institute of Kosovo and are presented in Figure 1. The surface reflection factor is assumed to be 0.20. July shows the highest values of monthly mean solar radiation; while minimum values are in December.

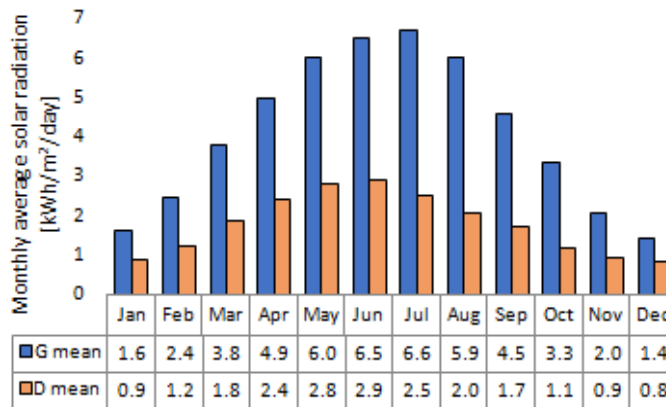


Figure 1. Monthly means daily global radiation G_m and diffuse radiation D_m on horizontal plane of Pristina.

In Figure 2 presents the yearly average of global radiation, horizontal plane, in Kosovo. As per Figure 1, the highest value of average global solar radiation in Kosovo is in the city of Gjakova. In Figure 3 is presented the annual average direct normal radiation, horizontal plane, for Kosovo cities.



Figure 2 Global horizontal radiation in Kosovo. Figure 3 Direct normal radiation in Kosovo

Source: <http://globalsolaratlas.info/downloads/kosovo>

III. ESTIMATION OF SOLAR RADIATION ON THE INCLINED SURFACE

Global solar radiation monthly mean data for an inclined surface are necessary for the designing of solar energy systems. These data are often not available. Therefore, we need tend to estimate the data by considering the monthly average of the global solar radiation on the horizontal plane (presented in Figure 3.), which is the most important parameter for estimating the optimum tilt angle.

The study presents a simple and universal method of determining the mean monthly global radiation based on the existing methodology given by Surface *Meteorology and Solar Energy* (SSE)

(2016) [34]. The total solar energy received on an inclined surface is “the sum of the beam, diffuse, and reflected radiation” [35], [36], [37].

Thus, the total solar radiation monthly average (in kWh/m²/day) for an inclined surface is calculated as follows:

$$\bar{H}_T = \bar{H}_B + \bar{H}_D + \bar{H}_R \quad (1)$$

Mathematically, the optimal value of the tilt angle (β_{opt}) is determined by differencing Eq. 1 depending on the angle of inclination (β).

$$\frac{d}{d\beta}(\bar{H}_T) = 0 \quad (2)$$

$$\bar{H}_T = \bar{R} \cdot \bar{H} \quad (3)$$

\bar{R} - Presents the ratio between the global radiation monthly mean on an inclined surface than for the horizontal one. \bar{R} ratio, defined by Bakirci [9], Liu and Jordan [35]:

$$\bar{R} = \left(1 - \frac{\bar{H}_D}{\bar{H}}\right) \bar{R}_b + \frac{\bar{H}_D}{\bar{H}} \left(\frac{1 + \cos \beta}{2}\right) + \rho \left(\frac{1 - \cos \beta}{2}\right) \quad (4)$$

\bar{H} and \bar{H}_d are the monthly average daily global and diffuse radiations on a horizontal surface; \bar{R}_b is the ratio of the mean daily direct radiation on an inclined surface to that on a horizontal surface, and the parameters correlated by ground reflection coefficient (albedo $\rho=0.2$); β represents the tilt angle. Radiation beam falling on an inclined surface is given by Liu and Jordan [35]:

$$\bar{H}_B = (\bar{H} - \bar{H}_d) \cdot \bar{R}_b \quad (5)$$

The reflected radiation is “the part of the total solar radiation that is reflected by the surface of the earth, and by any other surface intercepting object such as trees, terrain or buildings on to a surface exposed to the sky is termed as ground reflected radiation” [36]. Reflected radiation on an inclined surface is given by:

$$\bar{H}_R = \bar{H} \cdot \bar{R}_r \quad (6)$$

\bar{R}_r is the reflected conversion factor:

$$\bar{R}_r = \rho \left(\frac{1 - \cos \beta}{2}\right) \quad (7)$$

Where ρ is the constant that depends on the type of ground surrounding tilted surfaces and is called the ground reflectance or albedo. The value of albedo most commonly used is $\rho= 0.2$ for hot and humid tropical locations, $\rho= 0.5$ for dry tropical locations, and $\rho= 0.9$ for snow covered ground [38]. The ground reflection coefficient is assumed 0.2 for the climate conditions in Pristina.

Diffused radiation (\bar{H}_D) is the fraction of total solar radiation, received from the sun when its direction has been changed by atmospheric scattering [39]. The diffused radiation direction θ is highly variable and difficult to determine. It is a function of the condition of cloudiness and atmospheric clearness which is extremely unpredictable. Diffused radiation fraction is the sum isotropic, circumsolar, and horizon brightening components. The isotropic diffuse radiation component is “received evenly from the entire sky dome” [40]. The circumsolar diffuse part is received from the onward solar radiation dispersion and focused around the sun sky section [40]. The

horizon brightening component is concentrated near the horizon and it is most visible during clear skies [41]. In general, isotropic, circumsolar, and horizon brightening factors compose the diffuse fraction of radiation on an inclined surface. In Liu and Jordan [42] model, the diffuse radiation was assumed isotropic only while circumsolar and horizon brightening were taken as zero.

Diffuse radiation falling on an inclined surface is calculated as follows:

$$\bar{H}_D = \bar{H}_d \cdot \bar{R}_d \quad (8)$$

Where \bar{R}_d is the diffuse conversion factor presenting the ratio of diffuse solar radiation on an inclined surface to diffuse solar radiation on a horizontal surface, given as:

$$\bar{R}_d = \left(\frac{1 + \cos \beta}{2} \right) \quad (9)$$

The monthly average data of the daily global radiation, taken from Figure 3 are used in the following expressions for determination of other parameters. The following expression determines the monthly average of daily diffuse radiation [34]:

$$\begin{aligned} \frac{\bar{H}_d}{\bar{H}} &= 0.96268 - (1.45200 \cdot K_T) + (0.27365 \cdot K_T^2) \\ &+ (0.04279 \cdot K_T^3) + (0.000246 \cdot (SSHA)) \\ &+ (0.001189 \cdot (NHSA)) \end{aligned} \quad (10)$$

Where SSHA is the sunset hour angle in degrees on the “monthly average day (n)”, which can be calculated from the following equation:

$$SSHA = \omega_s = \cos^{-1}(-\tan(\phi - \delta) \tan \delta) \quad (11)$$

NHSA is the noon solar angle from the horizon in degrees on the “monthly average day (n)” and can be estimated as follows:

$$NHSA = 90^\circ - |\phi - \delta| \quad (12)$$

The monthly average clearness index \bar{K}_T is the ratio of monthly average daily radiation on a horizontal surface to the monthly average daily extraterrestrial radiation and can be obtained from:

$$\bar{K}_T = \bar{H} / \bar{H}_0 \quad (13)$$

Where, \bar{H}_0 is the monthly mean daily extraterrestrial radiation on a horizontal surface, which is calculated using [43]:

$$\begin{aligned} \bar{H}_0 &= \frac{24 \times 3600}{\pi} I_0 \left(1 + 0.033 \cos \left(\frac{360n}{365} \right) \right) \\ &\left(\cos \phi \cos \delta \sin \omega_s + \frac{2\pi\omega_s}{360} \sin \phi \sin \delta \right) \end{aligned} \quad (14)$$

Where I_0 represents the solar constant (1367 W/m^2); n shows the number of daily readings in a given month, counted from 1 January (1–365); ϕ is location geographic latitude; ω_s is sunrise/sunset angle on a horizontal surface; δ is sun declination. The “monthly average day” is the day of the month, whose solar declination is closest to the average declination for that month [44]. The maximum value of the declination angle (δ) is $+23.45^\circ$ during June 21st–22nd, while its minimum

value is -23.45° during December 20th–21st. The declination value is zero on March 22nd and September 22nd of the year, see Table 2.

Table 2. Monthly representative day and its corresponding declination (δ in degrees)

Month	Jan.	Feb.	Mar.	Apr.	May	June
Day (n)	17	47	75	105	135	162
Declination (δ)	-20.92	-12.05	-2.43	9.41	18.79	23.09
Month	July	Aug.	Sep.	Oct.	Nov.	Dec.
Day (n)	198	228	258	288	318	344
Declination (δ)	21.18	13.46	2.22	-9.60	-18.91	-23.05

According to Cooper (1969), declination angle is calculated using the following relation:

$$\delta = 23.45 \sin \left[360 \frac{(284+n)}{365} \right] \quad (15)$$

where n is the day of the year.

Various researchers have proposed numerous models which are classified as isotropic models and anisotropic to estimate solar radiation on inclined surfaces [41], [45], [46], [47], [48], [49], [50], [51], [52]. However, measured diffuse solar radiation data gives better estimates of the optimum tilt angle [30]. In this study, measured data of global and diffuse solar radiation has been utilized to calculate the optimum tilt angles. Liu and Jordan [41] (isotropic) model has been used to estimate total solar radiation on inclined flat surfaces facing south.

The angle of incidence for a surface oriented in any direction can be mathematically expressed by following relation [53]:

$$\begin{aligned} \cos \theta = & \sin \delta \sin \varphi \cos \beta - \sin \delta \cos \varphi \sin \beta \cos \gamma \\ & + \cos \delta \cos \varphi \cos \beta \cos \omega + \cos \delta \sin \varphi \sin \beta \cos \gamma \cos \omega \\ & + \cos \delta \sin \beta \sin \gamma \sin \omega \end{aligned} \quad (16)$$

For a surface in northern hemisphere facing south (i.e. $\gamma = 0$) Eq. (16) can be simplified as:

$$\cos \theta = \sin(\phi - \beta) \sin \delta + \cos(\phi - \beta) \cos \delta \cos \omega \quad (17)$$

For a horizontal surface ($\beta=0$), the angle of incidence (θ) becomes equal to zenith angle (θ_z). Substituting this value in Eq. (17), zenith angle can be written as:

$$\cos \theta_z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos \omega \quad (18)$$

The total solar energy received on an inclined surface is the sum of beam and diffuse radiations directly incident on a surface and reflected radiations (reflected by the surroundings).

According to Liu and Jordan (1960), the beam conversion factor \bar{R}_b is given as:

$$\bar{R}_b = \frac{\cos(\phi - \beta) \cos \delta \sin \omega + (\pi/180) \omega \sin(\phi - \beta) \sin \delta}{\cos \phi \cos \delta \sin \omega_s + (\pi/180) \omega_s \sin \phi \sin \delta} \quad (19)$$

Where ω is the sunrise (or sunset) hour angle for the inclined surface.

$$\omega = \min\{\omega_s, \omega'_s\} \quad (20)$$

The hour angles at sunrise and sunset ω_s are very useful parameters. Since, numerically these two angles have the same value the sunrise angle is negative, and the sunset angle is positive. Both can be calculated from the following expression:

$$\omega_s = \cos^{-1}(-\tan\phi \tan\delta) \quad (21)$$

If a surface is inclined from the horizontal, the Sun may rise over its edge after it has risen over the horizon. Therefore, the surface may shade itself for some days. The sunrise and sunset angles for an inclined surface (α) facing the equator (i.e. facing south for the northern hemisphere) are given by:

$$\omega_s' = \cos^{-1}(\tan(\phi - \beta) \tan\delta) \quad (22)$$

IV. RESULTS AND DISCUSSION

Equations (1-22) are used to determine the monthly mean daily global solar radiation on the south-facing inclined surface for the location under study, Pristina. By changing the tilt angle from 0° to 90° in values of 0.1° , the optimal tilt angle is defined by the corresponding value of maximum solar radiation for a certain period.

Based on the methodology used in Section 3 and on the Liu and Jordan model [41], estimations have been made to obtain the optimum monthly, seasonal, and yearly/annual tilt angles by corresponding to global solar radiation on an inclined surface – results are given in Table 3.

To continue with, the daily extraterrestrial radiation on a horizontal surface \bar{H}_0 , clearness index \bar{K}_T , diffuse solar radiation on a horizontal plane \bar{H}_d , optimal tilt angle β_{opt} , monthly average daily global radiation in optimal tilt angle \bar{H}_T , and the comparison \bar{H} with \bar{H}_T in percentage, has also been estimated and are presented in table 4, – which shows the minimum and maximum value of \bar{H}_T , that correspond to December and July respectively.

Table 3. The optimum monthly, seasonal, and annual tilt angles β_{opt} ($^\circ$) for Pristina

Station	Period	Month					
		Jan.	Feb.	Mar.	Apr.	May	Jun.
Pristina $\phi=42^\circ 39' N$	Monthly	64.8	56.5	430	25	9.2	0.3
	Seasonal	62.1	62.1	25.7	25.7	25.7	8.9
	Annual	34.7	34.7	34.7	34.7	34.7	34.7
Station	Period	Month					
		July	Aug.	Sep.	Oct.	Nov.	Dec.
Pristina $\phi=42^\circ 39' N$	Monthly	4.8	19.2	37.3	54.3	63.8	66.4
	Seasonal	8.9	8.9	50.9	50.9	50.9	62.1
	Annual	34.7	34.7	34.7	34.7	34.7	34.7

For all the months and different tilt angles, monthly average total solar radiation was calculated using Eq. (1). The findings have been plotted and are shown in Figure 4 for Pristina, where the tilt angle has been varied in the range of $0-90^\circ$ (in steps of 10°). Figure 4.a) and 4.b) show the total solar radiation versus tilt angle from January to June and from July to December respectively. It is apparent from the Figure 4 that the solar radiation is an intensive function of tilt angle. The calculated solar energy radiation incident on the flat surface is increased, with the

increasing of horizontal position from 0° to an angle of inclination, but a further increase of the tilt angle of the flat surface will result in the decreasing of solar radiation received.

Table 4. \bar{H}_o (kWh/m²/day), \bar{K}_T , \bar{H}_d (kWh/m²/day), β_{opt} (°), \bar{H}_T (kWh/m²/day) and comparison with \bar{H} in location **Pristina city.**

Month	\bar{H}_o	\bar{K}_T	\bar{H}_d	β_{opt} (°)	\bar{H}_T	\bar{H}_o / \bar{H}_T (%)
January	3.60	0.45	0.69	64.8	3.14	95.03
February	5.04	0.49	0.96	56.5	3.89	58.13
March	7.24	0.52	1.36	43	4.81	26.58
April	9.17	0.54	1.76	25	5.33	7.24
May	11.34	0.53	2.29	9.2	6.07	0.83
June	11.62	0.56	2.30	0.3	6.52	0.00
July	11.98	0.56	2.36	4.8	6.70	0.15
August	10.32	0.58	1.90	19.2	6.23	4.01
September	7.84	0.58	1.36	37.3	5.46	19.21
October	5.86	0.57	1.00	54.3	5.12	53.75
November	4.14	0.50	0.77	63.8	3.97	92.72
December	3.38	0.42	0.65	66.4	2.90	104.23
Average	7.63	0.53	1.45	-	5.01	21.34
% Gain	-	-	-	-	21.35	-

The result also indicated that the optimum angle differs with the yearly months. The maximum solar radiation is achieved for every month with unique optimum tilt angle – which increases in winter months and decreases to the minimum value in summer & autumn months, (see Figure 5). Figure 6 compares the average annual solar radiation on a tilted surface fixed at monthly, seasonal, and yearly optimum tilt angles for Pristina. As observed from the Figure 6, the annual average total solar radiation estimated at monthly optimum tilt angle is found to be the highest, followed by average annual solar radiation estimated at seasonal and annual optimum tilt angles value. The aforementioned optimal tilt angles are also plotted in Figure 5.

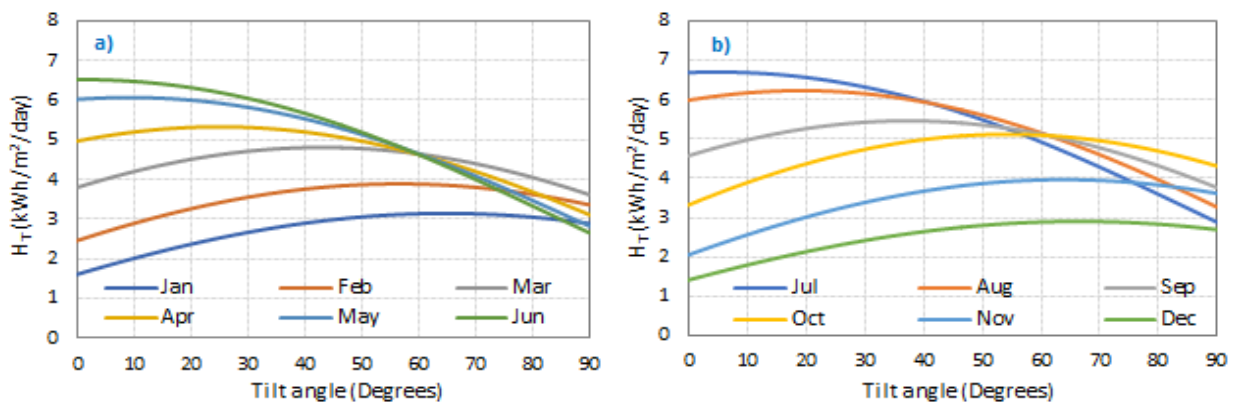


Figure 4. Monthly mean solar radiation data for Pristina when the tilt angle varieties from 0° to 90°:
 a) January-June and b) July-December.

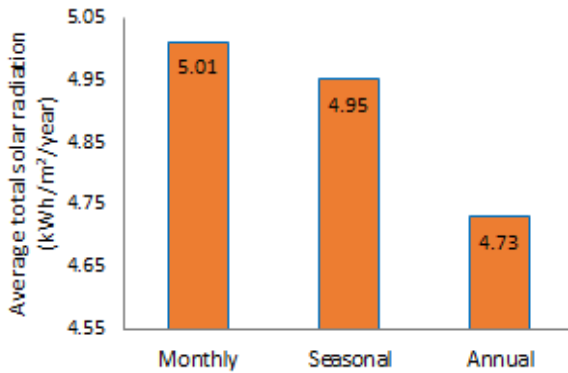


Figure 5. Average maximum total solar optimum annual for Pristina.

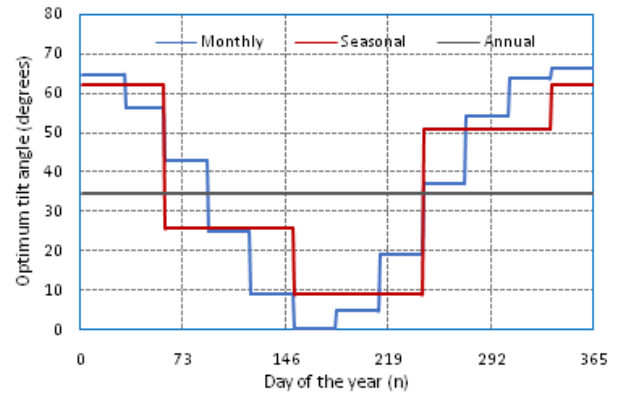


Figure 6. Variations of monthly, seasonal & annual optimum tilt angles for south-facing surface in Pristina.

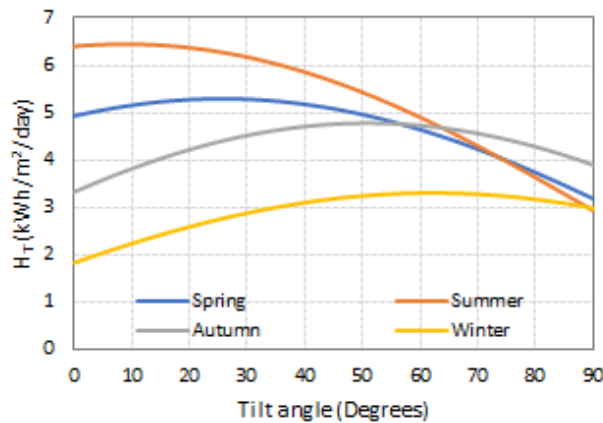


Figure 7. Seasonal mean solar radiation in Pristina for: Spring, Summer, Autumn, Winter, when the tilt angle charges from 0o to 90°. (Considering the azimuth angle=0o for maximum solar radiation).

Gains of solar radiation available on an inclined surface are defined as follows:

$$Gains (\%) = \left(\frac{\bar{H}_T (\beta = \beta_{opt,i})}{\bar{H}_T (\beta = 0)} - 1 \right) \times 100 \quad (23)$$

Where i = monthly, seasonal and annual.

Therefore, four seasonal optimum tilt angles were obtained (corresponding to each season) 62.1° in winter, 25.7° in spring, 8.9° in summer, and 50.9° in autumn. Thus, winters have a higher value of optimum tilt angle, while summers observe a lower tilt angle value. Annual optimum tilt angle was calculated by using expressions from Section 3 and has resulted to be 34.7° for Pristina.

Annual average total gains of solar radiation on an optimally tilted angle of a surface in comparison to a horizontal surface are 21.35% (monthly optimum tilt angle), 19.98% (seasonal optimum tilt angle) and 14.43% (annual optimum tilt angle).

Results for the diffuse radiation on a horizontal plane; the yearly (annual) and seasonal optimal tilt angles for each season; the monthly average daily global radiation for optimal tilt angle; and the comparison between H with H_T in percentage are presented in Table 5.

Table 5. \bar{H}_d (kWh/m²/day), β_{opt} (°), \bar{H}_T (kWh/m²/day) and the comparison with \bar{H} of Pristina for seasonal and annual tilt angles.

Month	Seasons	\bar{H}_d	Seasonal	Annual	Seasonal	Annual	\bar{H}_T / \bar{H}	\bar{H}_T / \bar{H}
January	Winter	0.69	62.1	34.7	3.14	2.78	95.03	72.67
February		0.96	62.1	34.7	3.88	3.66	57.72	48.78
March	Spring	1.36	25.7	34.7	4.64	4.77	22.11	25.53
April		1.76	25.7	34.7	5.33	5.27	7.24	6.04
May		2.26	25.7	34.7	5.91	5.70	-1.83	-5.32
June	Summer	2.29	8.9	34.7	6.48	5.88	-0.61	-9.82
July		2.35	8.9	34.7	6.69	6.17	0.00	-7.77
August		1.90	8.9	34.7	6.16	6.07	2.84	1.34
September	Autumn	1.36	50.9	34.7	5.34	5.46	16.59	19.21
October		1.01	50.9	34.7	5.11	4.87	53.45	46.25
November		0.76	50.9	34.7	3.88	3.54	88.35	71.84
December	Winter	0.65	62.1	34.7	2.90	2.54	104.23	78.87
Average		1.45	-	-	4.95	4.73	19.98	14.43
% Gain		-	-	-	19.98	14.43	-	-

For all the months and different tilt angles, monthly average total solar radiations for the city of Pristina, were calculated using the procedure described in the previous section, based on the Liu & Jordan model, estimates have been made to obtain optimum tilt angles and corresponding total solar radiation on inclined surfaces.

Figure 8 describes the variation of beam conversion factor with days of the year at various tilt angles for the city of Pristina. Also, Figure 9 shows the variation of diffuse and reflected conversion factors at different tilt angles.

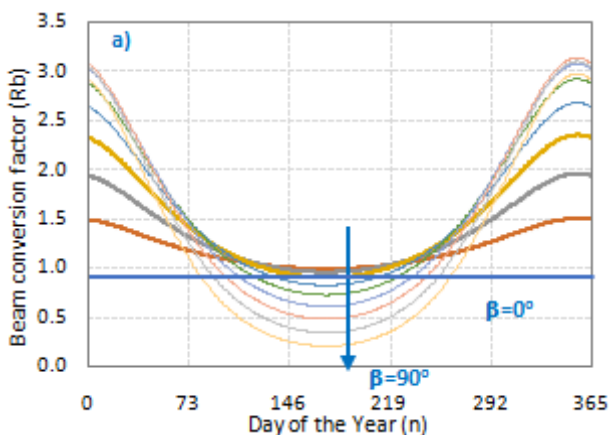


Figure 8. Beam conversion factor at different tilt angles for Pristina.

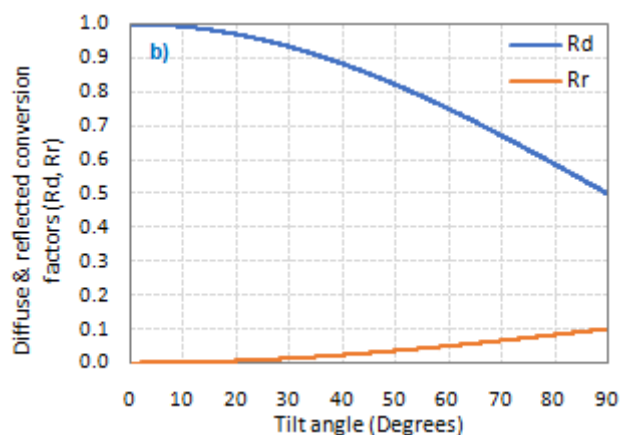


Figure 9. Diffuse and reflected conversion factors for Pristina.

V. CONCLUSION

By making adjustments in the surface inclination, energy falling on a surface can be significantly improved. Different months have different values for optimum tilt angles due to the change of the Sun's position. The following conclusions are drawn:

- Seasonal optimum tilt angles are 62.1° (winter), 25.7° (spring), 8.9° (summer), and 50.9° (autumn). Tilt angles are higher in winter rather than in summer.
- The Annual optimum tilt angle (β_{opt}) is 34.7°.
- Maximum and minimum values of total solar radiation at annual optimum tilt angle are 6.17 kWh/m²/day (July) and 2.54 kWh/m²/day (December).

It is recommended that where possible the surface must be inclined at monthly or seasonal optimum tilt angles to reach the highest possible values of solar radiation. The annual optimum tilt angle can be used for surfaces with fixed inclination where the changing of tilt angle is not often possible.

By doing so, the system optimization is achieved to enable the highest absorption of solar radiation from an inclined surface.

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